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Abstract

Over the last decade a new consensus model has emerged in monetary macroeconomics, labelled New Keynesian macroeconomics (Clarida et al., 1999). It consists of three simple building blocs: a forward-looking IS-equation that is derived from the optimization problem of a representative household, a forward-looking Phillips curve that maps the optimal pricing decisions of monopolistically competitive firms facing restrictions on their ability to adjust wages or prices in a flexible manner, and a relationship that describes how monetary policy is conducted. In Bofinger, Mayer and Wollmershäuser (2002a, 2002b) we developed the BMW model which takes this standard dynamic macro model to an intermediate audience in a down-to-earth fashion. This paper presents the linkages between our static BMW approach and a dynamic New Keynesian macro model.

Keywords: BMW model, New Keynesian macroeconomic model, optimal monetary policy

JEL classification: A 20, E10, E52, E58, F41

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1 Introduction

Over the last decade a new consensus model has started to emerge which is commonly used to evaluate and discuss systematic monetary policy (see for example Clarida et al., 1999). This so-called New Keynesian macro model shares a very simple dynamic structure that is centered around three building blocs: an intertemporal IS-curve which can be derived from the optimization behaviour of households allocating among others consumption optimally over time, a forward-looking aggregate supply relationship labeled New Keynesian Phillips curve which maps the optimal pricing decisions of firms being faced with restrictions to adjust prices or wages in a flexible manner, and a relationship depicting the way according to which monetary policy is conducted.

Up to now there are only a view approaches available that try to present the new consensus model to an intermediate audience (Romer, 2000, Walsh, 2002). In Bofinger, Mayer and Wollmershäuser (2002a, 2002b) we also presented an algebraic and graphical treatment of a static version of the New Keynesian macro model in which we preserved many of its main insights. Despite its simplicity this so-called BMW model can deal with questions like flexible inflation targeting, simple monetary policy rules, stability and central bank credibility at an intermediate macro level.

The purpose of the present paper is to demonstrate the proximity of the static BMW approach to a dynamic New Keynesian macro model. It is structured as follows: In Section 2 we shortly summarize the mathematical foundations of our static approach by deriving the key relationships from a straightforward Lagragian approach. In Section 3 we illustrate the mathematical backbone of the standard New Keynesian macro model that was originally proposed by Clarida, Gali and Gertler (1999). In Section 4 we will then demonstrate that the BMW model can be viewed as a static approximation of this New Keynesian macro model. We show under which assumptions the impulse response functions and the efficient frontiers of the two models converge.

2 Optimal monetary policy in the BMW model

The BMW model consists of two structural equations:

(2.1) $$y = a - br + \varepsilon_1$$

(2.2) $$\pi = \pi_0 + dy + \varepsilon_2 .$$

Equation (2.1) is an IS equation according to which the output gap y depends on autonomous demand components a, on the real interest rate r and on a demand shock ε_1. Equation (2.2) represents a Phillips curve. Deviations of the inflation rate π from its medium term target level π_0 occur in response to movements in the output gap and to supply shocks ε_2. In contrast to standard expectations-augmented Phillips curves (see for example Romer, 2001, Chapter 5) we replaced the expected inflation rate by the inflation target π_0. The rationale behind this is that we assume that the central bank possesses a high degree of credibility, so that the private sector's inflation expectations are identical with the central bank's inflation target. All the structural parameters of the model are assumed to be positive, and the disturbances ε_1 and ε_2 are i.i.d. random variables with zero mean and variances $\sigma^2_{\varepsilon 1}$ and $\sigma^2_{\varepsilon 2}$ respectively.

The policy instrument of the central bank is the real interest rate r. If the central bank pursues an optimum intererst rate policy, it sets its interest rates so as to minimize a loss function L

(2.3) $$L = \left(\pi - \pi_0\right)^2 + \lambda y^2$$

that sums up deviations of inflation from its target level and deviations of output from potential (i.e. non-zero output gaps). The parameter λ represents the preferences of the central bank with respect to output stabilization.

The optimization problem of the central bank is solved in two steps. First, it is important to recognize that the central bank directly influences the output gap via equation (2.1) and that it only indirectly influences inflation through its impact on the output gap, and hence via equation (2.2). From this follows, second, that is suffices – from the point of view of optimal control theory – to consider the output gap as 'policy variable' and to determine the optimum output gap. In other words, the use of the instrument itself is not associated with any real costs so that the loss function is minimized subject to the Phillips curve only. Thus, we get the following Lagrangian:

(2.4) $$H = \left[\left(\pi - \pi_0 \right)^2 + \lambda y^2 \right] + \xi \left(\pi - \pi_0 - dy - \varepsilon_2 \right).$$

The Lagrange multiplier ξ measures the costs if the central deviates from its optimal solution. Taking the derivative with respect to the output gap and the inflation rate we arrive at the following two first order conditions:

(2.5) $$\xi = \frac{2\lambda y}{d}$$

(2.6) $$\xi = -2\left(\pi - \pi_0 \right).$$

Eliminating the Larange multiplier ξ and solving the resulting expression for π gives the consolidated foc:

(2.7) $$\pi = \pi_0 - \frac{\lambda y}{d}.$$

Following Svensson and Woodford (2003) we can give a first intuition of inflation targeting to macroeconomic students at the intermediate level by equation (2.7) which is the so-called targeting rule of the central bank. A targeting rule gives a high level specification of monetary policy, that is directly derived from the central bank's strategy. Therefore it can be characterised as a high level specification of monetary policy and is well in line with the current institutional environment of leading inflation targeters that aims at committing the central bank at the target level. Given the structure of the model it is the task of the central bank to control the output gap in such a way that equation (2.7) will hold with equality in the absence of factors beyond the control of monetary policy (e.g., shocks to the market for borrowed reserves).

The optimum output gap consistent with the targeting rule of the central bank is obtained by equating (2.7) and (2.2) and by solving the resulting equation for y:

(2.8) $$y = -\frac{d}{\left(d^2 + \lambda \right)} \varepsilon_2 .$$

After replacing y in equation (2.7) with equation (2.8), we get a reduced form of the inflation rate:

(2.9) $$\pi = \pi_0 + \frac{\lambda}{\left(d^2 + \lambda\right)} \varepsilon_2 .$$

The optimal interest rate can finally be represented as a function of the exogenous variables ε_1 and ε_2 by inserting equation (2.8) into equation (2.1) and by solving the resulting expression for the instrument rule:

(2.10) $$r = \frac{a}{b} + \frac{1}{b}\varepsilon_1 + \frac{d}{b\left(d^2 + \lambda\right)} \varepsilon_2 .$$

The instrument rule is a low level specification of monetary policy as it is not specific to the central bank's strategy. Hence by only looking at the instrument rule it is hard to guess which specific strategy the central bank follows. Additionally in real live no central bank has yet been committed to an instrument rule despite its popularity in academic literature. As noted by Svensson (2003) it is not necessary to specify the instrument rule explicitly as the central bank's strategy is suffiently described by its loss function and the associated targeting rule. Nevertheless instrument rules have its virtues as they directly specify the reaction of monetary policy to the exogenous supply and demand disturbances. This optimal monetary policy rule at the instrument level is described by the following characteristics:

- The optimal response to demand shocks ε_1 does not depend on preferences λ. Therefore, each preference type λ adjusts the real interest rate according to $r = \left(1/b\right)\varepsilon_1$ which fully closes the initial output gap.
- The reaction of the central bank to supply shocks depends on preferences λ. A central bank that only cares about inflation ($\lambda = 0$), requires a strong real rate increase and accordingly a high output loss. With an increasing λ the interest rate response weakens and accordingly the output loss decreases whereas the inflation loss increases.

- In equilibrium ($\pi = \pi_0, y = 0$) the real interest rate will be given by $r_0 = a/b$. In line with Blinder (1998, p. 31) this rate can be regarded as a neutral real short-term interest rate.

3 Optimal monetary policy in the New Keynesian model

The New Keynesian macroeconomic model is a simple dynamic equilibrium model which nests a forward-looking IS curve (3.1) and a forward-looking Phillips curve (3.2) with nominal rigidities. These two equations are the backbone of a sticky price model (see e.g. Clarida et al., 1999). As both behavioral equations evolve explicitly from optimizing households and firms, current economic behavior depends critically on expectations of the future course of monetary policy, as described by the expected path of future short term interest rates:

$$(3.1) \qquad y_t = E_t y_{t+1} - b\left[\left(i_t - E_t \pi_{t+1}\right) - r_0\right] + \varepsilon_{1,t}$$

$$(3.2) \qquad \pi_t = \delta E_t \pi_{t+1} + d y_t + \varepsilon_{2,t}\,.$$

In the IS equation (3.1) $E_t y_{t+1}$ denotes the expected output gap of the next period based on the information available in period t. According to the Fisher equation the current real interest rate is defined as the difference between the current nominal short-term interest rate i_t and the expected inflation rate over the next period $E_t \pi_{t+1}$. r_0 is the average natural real rate of interest which is consistent with a zero output gap, and the term $\varepsilon_{1,t}$ denotes a demand shock. With $b > 0$ the output gap y_t is negatively related to deviations of the real interest rate from its natural rate. The Phillips curve (3.2) relates inflation π_t positively to the output gap y_t and the expected inflation rate of the next period based on information in the current period $E_t \pi_{t+1}$. The term $\varepsilon_{2,t}$ denotes a supply shock.

The two disturbance terms are assumed to follow a first-order autoregressive process

$$(3.3) \qquad \varepsilon_{1,t} = \rho_1 \varepsilon_{1,t-1} + \hat{\varepsilon}_{1,t}$$

$$(3.4) \qquad \varepsilon_{2,t} = \rho_2 \varepsilon_{2,t-1} + \hat{\varepsilon}_{2,t}$$

where $0 \leq \rho_1, \rho_2 \leq 1$ and where both $\hat{\varepsilon}_{1,t}$ and $\hat{\varepsilon}_{2,t}$ are i.i.d. random variables with zero mean and variances $\sigma^2_{\varepsilon 1}$ and $\sigma^2_{\varepsilon 2}$ respectively. These autocorrelated shocks serve as a substitute in simple New Keynesian models to mask omitted economic structure. In the IS relationsship endogeneous persistence can be implemented by assuming habit formation. This means that some fraction of households optimises while another fraction of households simply centres today's consumption decisions around last periods consumption level. This habit formation is usually referred to as 'rule-of-thumb behaviour' (Amato and Laubach Thomas, 2003). It virtually does not produce any computational costs as households do not need to optimize. Additionally rule of thumb setters learn as the last period's output gap incorporates information of those parts of households that optimised. Equally one can generate endogeneous persistence in the Phillips curve by introducing rule-of-thumb behaviour on some part of price setters (Christiano et al., 2001). Hence those economic agents that are not called upon to reset prices optimally simply update their prices by following a rule-of-thumb. In particular one may assume that some price setters update their prices by yesterday's inflation rate. Again rule-of-thumb setters implicitly learn as π_{t-1} incorporates the pricing decisions of those agents that optimised in the previous period.

The objective function of the central bank is an intertemporal loss function, summing up the expectations about discounted current and future deviations of inflation from target and output from potential:

$$(3.5) \qquad L_t = E_t\left[\sum_{\tau=0}^{\infty} \delta^{\tau}\left\{\left(\pi_{t+\tau} - \pi_0\right)^2 + \lambda y^2_{t+\tau}\right\}\right].$$

The parameter δ denotes the discount factor[1], and the parameter λ measures the weight policymakers attach to output stabilization relative to inflation stabilization.

For the solution of the central bank's dynamic optimization problem we adopted an approach which basically draws on Clarida et al. (1999) and Svensson (2003). For the reasons already outlined in Section 2, the intertemporal loss function (3.5) is minimized subject to the Phillips curve equation. This leads to the following Lagrangian

[1] Woodford (2003, Chapter 6) showed that this form of intertemporal loss function can be derived as a quadratic approximation to (the negative) expected utility of the representative household in the same optimizing sticky-price model as is used to derive structural equations (3.1) and (3.2).

$$H_t = \sum_{\tau=0}^{\infty} \delta^{\tau} \left\{ \left[\left(\pi_{t+\tau,t} - \pi_0 \right)^2 + \lambda y^2_{t+\tau,t} \right] + \xi_{t+\tau,t} \left(\pi_{t+\tau,t} - \delta \pi_{t+\tau+1,t} - d y_{t+\tau,t} \right) \right\} \tag{3.6}$$

where $x_{t+\tau,t}$ denotes the τ-period-ahead expectations of variable x, conditional on the central bank's information in period t on the state of the economy and the transmission mechanism of monetary policy (which is equal to $E_t x_{t+\tau}$). The term in parantheses following the dynamic Lagrange multiplier $\xi_{t+\tau,t}$ represents the central bank's τ-period-ahead forecast of equation (3.2) in period t. Differentiating with respect to $\pi_{t+\tau,t}$ and $y_{t+\tau,t}$ gives the two first-order conditions

$$\xi_{t+\tau,t} = -2\left(\pi_{t+\tau,t} - \pi_0 \right) \tag{3.7}$$

$$\xi_{t+\tau,t} = \frac{2\lambda y_{t+\tau,t}}{d}. \tag{3.8}$$

A basic assumption underlying the first foc is that the central bank takes private sector expectations about next period inflation rate $\pi_{t+\tau+1,t}$ as given. The literature typically refers to this kind of procedure as discretionary optimization, in contrast to optimization under commitment.[2] Setting $\tau = 0$ and eliminating the Lagrange multiplier leads to the consolidated foc:

$$y_t = -\frac{d}{\lambda}\left(\pi_t - \pi_0 \right). \tag{3.9}$$

Obviously the targeting rule of the central bank is identical to relationship (2.7) which was derived in the static BMW model. Henceforth optimal monetary policy is conducted in an identical fashion. Inserting (3.9) into (3.2) yields the following forward-looking first-order difference equation

$$\frac{\lambda + d^2}{\delta\lambda} \pi_t = E_t \pi_{t+1} + \frac{d^2}{\delta\lambda} \pi_0 + \frac{1}{\delta} \varepsilon_{2,t} \tag{3.10}$$

which can be solved using the MSV (minimal set of state variables) approach of McCallum (1983) (see Appendix A for details). The dynamics of the inflation rate then follow

$$(3.11)\qquad \pi_t = \frac{d^2}{d^2+\lambda-\delta\lambda}\pi_0 + \frac{\lambda}{d^2+\lambda-\delta\lambda\rho_2}\varepsilon_{2,t}.$$

Taking one-period-ahead expectations of (3.11) (and considering equation (3.4)) gives

$$(3.12)\qquad E_t\pi_{t+1} = \frac{d^2}{d^2+\lambda-\delta\lambda}\pi_0 + \frac{\lambda\rho_2}{d^2+\lambda-\delta\lambda\rho_2}\varepsilon_{2,t}.$$

Inserting (3.11) into the consolidated foc (3.9) yields the dynamic law of motion of the output gap:

$$(3.13)\qquad y_t = d\left(\frac{1-\delta}{d^2+\lambda-\delta\lambda}\right)\pi_0 - \frac{d}{d^2+\lambda-\delta\lambda\rho_2}\varepsilon_{2,t}.$$

Taking again one-period-ahead expectations of (3.13) (and considering equation (3.4)) gives

$$(3.14)\qquad E_t y_{t+1} = d\left(\frac{1-\delta}{d^2+\lambda-\delta\lambda}\right)\pi_0 - \frac{d\rho_2}{d^2+\lambda-\delta\lambda\rho_2}\varepsilon_{2,t}.$$

With the dynamics of inflation and output at hand we can finally derive the optimal interest rate rule. Inserting (3.12), (3.13) and (3.14) into the aggregate demand equation (3.1) and solving the resulting expression for the monetary policy instrument i_t yields the following instrument rule:

$$(3.15)\qquad i_t = r_0 + \frac{d^2}{d^2+\lambda-\delta\lambda}\pi_0 + \frac{1}{b}\varepsilon_{1,t} + \frac{b\lambda\rho_2 - d\rho_2 + d}{b\left(d^2+\lambda-\delta\lambda\rho_2\right)}\varepsilon_{2,t}.$$

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[2] If a central bank credibly commits to a once-and-for-all policy rule, it internalizes the effects of its own interest rate decision on the expectations of the private sector. For $\tau \geq 1$ the first foc would then be $2\delta^\tau\left(\pi_{t+\tau,t} - \pi_0\right) - \delta^\tau\xi_{t+\tau,t} - \delta^{\tau-1}\delta\xi_{t+\tau-1,t} = 0$.

If a central bank follows this rule

- it perfectly offsets demand shocks $\varepsilon_{1,t}$ as the interest rate impacts on the output gap with a factor b;
- it faces a trade-off in the case of supply shocks $\varepsilon_{2,t}$ which crucially depends on the preferences of the central bank λ;
- it keeps the nominal interest rate constant in the absence of shocks.

A basic requirement for ensuring the long-run neutrality of money is that δ approaches unity. From a theoretical point of view setting δ equal to unity is somewhat problematic as δ depicts the discount factor of a representative household that maximizes its utility. It can be shown that the neutral real interest rate r_0 is defined as $-\log(\delta)$. Thus, in order to avoid a value of r_0 equal to zero, δ must be below 1.[3] The reason why this discount factor also appears in the Phillips curve (3.2) is that profits of firms are assumed to be transferred to households so that prices are discounted with δ. From an empirical perspective the postulation that δ should be one is less problematic as estimated discount factors are typically not statistically different from one (Rotemberg and Woodford, 1999). In the case of $\delta = 1$, the long-run inflation rate and the long-run inflation expectations converge to the level of the inflation target ($\pi_t = E_t\pi_{t+1} = \pi_0$), the long-run output gap is zero ($y_t = 0$), and the long-run nominal interest rate equals the sum of the equilibrium real interest rate and the inflation target ($i_t = r_0 + \pi_0$). Otherwise there will be a long-run trade-off between the level of the inflation target (which can be freely chosen by the central bank) and the level of the output gap. To see this assume that $\delta < 1$, meaning that the costs resulting from the anticipation of deviations of inflation from its target level and of output from potential are weighted more strongly as they occur earlier in time. Inflation will then be biased downwards ($\pi_t < \pi_0$) at the expense of a positive output gap which crucially depends on the central bank's choice of π_0:

$$(3.16) \qquad y_t = d\left(\frac{1-\delta}{d^2 + \lambda - \delta\lambda}\right)\pi_0 > 0.$$

[3] Quarterly models often assume $\delta = 0.99$ (0.995), so $r_0 = 4.0$ % (2.0 %).

The point that the long-run Phillips curve is steep and not vertical was also made – among others – by Woodford (1999, p. 32).

4 Approximating the New Keynesian model by the BMW model

The dynamics of the New Keynesian model can be simplified substantially, if we specify two of the model's parameters. First, we set δ equal to one. This has the additional convenient effect that in the limit, after scaling the intertemporal loss function (3.5) by a factor $(1-\delta)$, the intertemporal loss approaches the weighted sum of the unconditional variances of inflation and the output gap (Svensson, 2003):

(4.1) $$\lim_{\delta \to 1}(1-\delta)L_t = \mathrm{Var}[\pi_t] + \lambda \mathrm{Var}[y_t].$$

By interpreting the intertemporal loss in terms of the variances of the goal variables, the optimality of an interest rate rule (such as (3.15)) can then be illustrated by the so-called efficient frontier which depicts the second-order trade-off between the variances of inflation and output (instead of the aforementioned trade-off between their levels which is avoided when $\delta = 1$)(Taylor, 1979). Hence although there is no trade off at the level of the variables, there is a trade off in the second moments that is compatible with the same steady state solution.

Second, we will gradually lower the autocorrelation of the supply shock ρ_2 to zero. This exercise is most crucial for the purpose of the present Section as ρ_2 turns out to be the exclusive source of dynamic movements in a simple New Keynesian macro model as originally proposed by Clarida et al. (1999).

4.1 The dynamics of the inflation rate

For $\delta = 1$ the dynamics of the inflation rate as expressed in equation (3.11) reduces to

(4.2) $$\pi_t = \pi_0 + \frac{\lambda}{d^2 + \lambda - \lambda \rho_2} \varepsilon_{2,t}.$$

According to (4.2) deviations of the inflation rate from its target only occur in the event of supply shocks. The extent of the deviation crucially depends on the preference parameter of the central bank, and hence on the extent to which the central bank accommodates supply shocks. By additionally setting $\rho_2 = 0$ equation (4.2) further reduces to

$$\pi_t = \pi_0 + \frac{\lambda}{d^2 + \lambda} \varepsilon_{2,t} \tag{4.3}$$

which is identical to equation (2.9) of the BMW model.

The expected inflation rate for the next period which was given by equation (3.12) can also be substantially simplified after inserting $\delta = 1$ and $\rho_2 = 0$:

$$E_t \pi_{t+1} = \pi_0 . \tag{4.4}$$

Equation (4.4) implies that in the long-run inflation is expected to be anchored by the central bank's inflation target. Recall that this was a basic simplification for the formulation of the Phillips curve in the BMW model. In Section 2 we justified equation (4.4) by the assumption that the central bank's monetary policy is credible and that the private sector therefore believes in the central bank's commitment to the inflation target. Now we provide the analytical proof of this simplification which is valid in a macroeconomic environment in which the duration of shocks is limited to one period.

4.2 The dynamics of the output gap

If we set δ to be 1, the non-neutrality of money in equation (3.13) disappears and the dynamics of the output gap evolve according to

$$y_t = -\frac{d}{d^2 + \lambda - \lambda \rho_2} \varepsilon_{2,t} . \tag{4.5}$$

As was the case with the inflation rate, deviations of output from potential only occur in response to supply shocks which are only partially compensated by the central bank. By setting $\delta = 1$ and $\rho_2 = 0$ equation (4.5) can be further simplified to

(4.6) $$y_t = -\frac{d}{d^2 + \lambda}\varepsilon_{2,t}$$

which is then identical to equation (2.8) of the BMW model.

4.3 The optimal interest rate rule

For $\delta = 1$ and $\rho_2 = 0$ the optimal interest rate rule of the dynamic New Keynesian model simplifies to

(4.7) $$i_t = r_0 + \pi_0 + \frac{1}{b}\varepsilon_{1,t} + \frac{d}{b\left(d^2 + \lambda\right)}\varepsilon_{2,t}.$$

With the nominal interest rate being defined as $i_t = r_t + E_t\pi_{t+1}$ and with equation (4.4), the policy rule can be expressed in terms of the real interest rate

(4.8) $$r_t = r_0 + \frac{1}{b}\varepsilon_{1,t} + \frac{d}{b\left(d^2 + \lambda\right)}\varepsilon_{2,t}$$

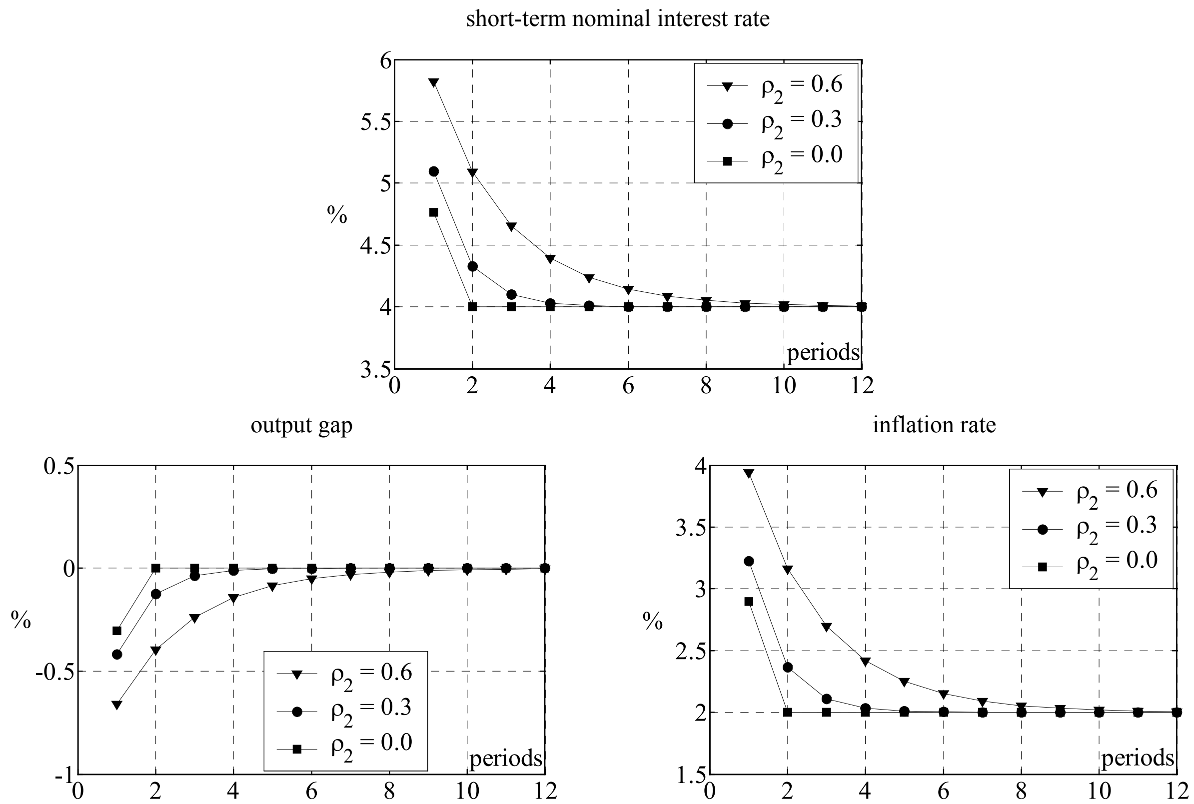
which is identical to the optimal policy rule (2.10) of the BMW model if the neutral real short-term interest rate r_0 equals a/b.

4.4 The dynamics of the two models

The dependence of the dynamic behavior of the New Keynesian model on the autocorrelation coefficient of the supply shock ρ_2 and its identity with the BMW model for $\delta = 1$ and $\rho_2 = 0$ can be illustrated by calculating and depicting the impulse response functions of the New Keynesian model. Figure 1 shows the responses of the nominal interest rate, the output gap and the inflation rate to a one standard deviation supply shock which hits the economy in period 1. For this simulation the model was calibrated as in Bofinger et al. (2002b): $b = 0.4$, $d = 0.34$, $\lambda = 1$, $\delta = 1$, $\mathrm{Var}\left[\varepsilon_{2,t}\right] = 1$, $\pi_0 = 2$, and $r_0 = 2$ (implying a value of a in the BMW model of 0.8). The basic message of Figure 1 is that the lower ρ_2, the lower the persistence of the deviation of

i_t, y_t, and π_t from their equilibrium levels 4 ($= r_0 + \pi_0$), 0, and 2 ($= \pi_0$), respectively. For $\rho_2 = 0$, the dynamics are reduced to a single peak in period 1 which is typical for a comparative static model – such as the BMW model – since in the period directly following the shock (period 2) the model's variables immediately return to their equilibrium values.

Figure 1: Responses to a supply shock



While the comparative statics appear to be plausible at first sight, the high initial jump and the gradual return of the variables that follows the jump for $\rho_2 > 0$ require a somewhat deeper look at the dynamics of the New Keynesian model. To explain this we take the Phillips curve as an example. Equation (3.2) not only produces a positive correlation between the level of inflation and real output, it also defines a negative correlation between the expected change in inflation and real output (for $\delta = 1$). The dynamic implication of these opposite-signed correlations is that, in response to, say, a positive shock to inflation, the level of inflation will rise, while the change in inflation will always be negative. This can only occur if inflation jumps up immediately in response to the shock, and subsequently falls back to its equilibrium.[4]

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[4] While the New Keynesian models is derived from sound economic principles, this dynamic implication is seriously at odds with the data. There is a host of empirical evidence suggesting that both inflation and output

4.5 The efficient frontier

The fact that the BMW model represents a special case of the New Keynesian model can also be demonstrated by computing the efficient frontier. On the basis of equations (4.2) and (4.5) the variances of inflation and output can be calcualted as

$$\text{(4.9)} \qquad \mathrm{Var}\left[\pi_t\right]=\left(\frac{\lambda}{d^2+\lambda-\lambda\rho_2}\right)^2 \mathrm{Var}\left[\varepsilon_{2,t}\right]$$

$$\text{(4.10)} \qquad \mathrm{Var}\left[y_t\right]=\left(\frac{d}{d^2+\lambda-\lambda\rho_2}\right)^2 \mathrm{Var}\left[\varepsilon_{2,t}\right].$$

Since $\varepsilon_{2,t}$ follows a first-order autoregressive process (see equation (3.4)), its variance can be expressed as

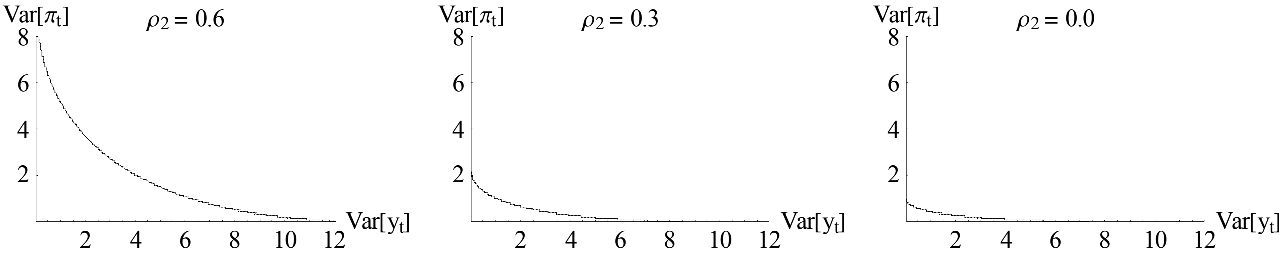
$$\text{(4.11)} \qquad \mathrm{Var}\left[\varepsilon_{2,t}\right]=\frac{\mathrm{Var}\left[\hat{\varepsilon}_{2,t}\right]}{1-\rho_2^2}.$$

The values of $\mathrm{Var}\left[y_t\right]$ and $\mathrm{Var}\left[\pi_t\right]$ that are associated with different values of λ are the plotted as the convex efficient frontier in Figure 2. At points on the frontiers, it is not possible for the policymakers to reduce the variance of inflation without increasing the variance of the output gap, given that the central bank sets interest rates according to the optimum policy rule (3.15). Policymakers can, however, choose alternative points along the frontier by varying the relative weight λ that they put on output versus inflation stabilization. For the construction of the curves we increased the preference parameter λ from 0.01 (high preference for inflation stabilization; the lower right end of the frontier) to 10 (a high preference for output stabilization; the upperleft end of the frontier) in steps of 0.01. With a falling ρ_2, both, $\mathrm{Var}\left[\varepsilon_{2,t}\right]$ and the squared term in brackets in equations (4.9) and (4.10) will become smaller so that the efficient frontier shifts towards the origin of the $\mathrm{Var}\left[y_t\right]-\mathrm{Var}\left[\pi_t\right]$ space. For $\rho_2=0$ the efficient frontier of the New

exhibit gradual and 'humpshaped' responses to real and monetary shocks, instead of the 'jump' behavior resulting from purely forward-looking model specifications (see e.g. Estrella and Fuhrer, 2002).

Keynesian model is identical to that of the BMW model which is shown by Figure 19 in Bofinger et al. (2002a).

Figure 2: Efficient frontiers



5 Conclusion

This paper, for the time being, terminates the BMW project. Within this project we have written five papers over the last 2 years that aim at bringing the basic insights of a new class of dynamic models labelled New Keynesian macroeconomics to an intermediate audience. The purpose of this final paper was to clarify the proximity of the BMW model to standard New Keynesian macro models à la Clarida, Gali and Gertler (1999). The key to understand this proximity is to see that under discretion the first-order conditions that govern the dynamics of the system are identical. Therefore, we showed that when demand and supply shocks converge from an first-order autoregressive process to a white noise process the 'dynamics' of the model (as encapsulated in the consolidated first order condition) become the same. To illustrate this point, we showed the convergence of the impulse response functions and the efficient frontiers.

The authors are sure that the IS/LM model has come into its ages and will be replaced gradually. We hope to equip those who have to teach monetary macroeconomics at an intermediate level with a powerful alternative that provides graphical and analytical devices in a unified approach.

A Solving the forward-looking first-order difference equation

A.1 The MSV approach

A very tedious way to solve a forward-looking first-order difference equation of the following type

(A.1) $$x_t = a + bE_t x_{t+1} + cz_t \text{ with}$$

(A.2) $$z_t = \rho_z z_{t-1} + \varepsilon_{z,t}$$

(where x_t is the endogenous variable, z_t is an exogenous variable, a, b, c and ρ_z are constant coefficients, and $\varepsilon_{z,t}$ is white noise) is to apply the procedure of forward iteration. The upshot of this procedure would be that x_t depends only on z_t (and the constants). McCallum (1983) therefore proposed an alternative solution procedure according to which it suffices to conjecture a solution of the difference equation that contains a minimal set of state variables (the so-called MSV approach). Specifically, he considered the following type of solution

(A.3) $$x_t = \alpha + \beta z_t$$

where the constants α and β are yet to be determined. Forming expectations of (A.3) (and considering (A.2)) yields

(A.4) $$E_t x_{t+1} = \alpha + \beta\rho_z z_t .$$

Inserting (A.4) into (A.1) gives

(A.5) $$x_t = \left(a + b\alpha\right) + \left(b\beta\rho_z + c\right) z_t .$$

By applying the method of undetermined coefficients, we obtain

(A.6) $$\alpha = \frac{a}{1-b}, \text{ and}$$

(A.7) $$\beta = \frac{c}{1-b\rho_z}.$$

Thus, a general solution to the forward-looking first-order difference equation (A.1) is

(A.8) $$x_t = \frac{a}{1-b} + \frac{c}{1-b\rho_z} z_t .$$

A.2 Applying the MSV approach to the New Keynesian model

The MSV approach can be applied to solve the forward-looking first-order difference equation for inflation which is given by (3.10):

(A.9) $$\frac{\lambda + d^2}{\delta\lambda}\pi_t = E_t\pi_{t+1} + \frac{d^2}{\delta\lambda}\pi_0 + \frac{1}{\delta}\varepsilon_{2,t} .$$

The shock term $\varepsilon_{2,t}$ follows the first-order autoregressive process specified by equation (3.4):

(A.10) $$\varepsilon_{2,t} = \rho_2\varepsilon_{2,t-1} + \hat{\varepsilon}_{2,t} .$$

In this system the minimal set of state variables includes only $\varepsilon_{2,t}$, so the solution will be of the form

(A.11) $$\pi_t = \alpha + \beta\varepsilon_{2,t} .$$

Taking expectations of (A.11),

(A.12) $$E_t\pi_{t+1} = \alpha + \beta\rho_2\varepsilon_{2,t} ,$$

inserting (A.12) into (A.9), and solving the resulting expression for π_t yields

(A.13) $$\pi_t = \left(\frac{\delta\lambda\alpha + d^2\pi_0}{\lambda + d^2}\right) + \left(\frac{\lambda + \beta\delta\lambda\rho_2}{\lambda + d^2}\right)\varepsilon_{2,t}\,.$$

Setting the first term in paranthesis equal to α and the second term in paranthesis equal to β, and solving the resulting equations for α and β, respectively, finally gives

(A.14) $$\alpha = \frac{d^2}{\lambda + d^2 - \delta\lambda}\pi_0\text{, and}$$

(A.15) $$\beta = \frac{\lambda}{\lambda + d^2 - \delta\lambda\rho_2}\,.$$

The solution of (A.9) then is

(A.16) $$\pi_t = \frac{d^2}{\lambda + d^2 - \delta\lambda}\pi_0 + \frac{\lambda}{\lambda + d^2 - \delta\lambda\rho_2}\varepsilon_{2,t}\,.$$

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